LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

B.Sc. DEGREE EXAMINATION – **STATISTICS**

FIRST SEMESTER – **APRIL 2023**

UMT 1303 – MATHEMATICS FOR STATISTICS

Date: 08-05-2023   Dept. No.   Max. : 100 Marks
Time: 09:00 AM - 12:00 NOON

SECTION A

Answer ALL the Questions

1.	**Definition**	**(5 x 1 = 5)**	
a)	Define the differential coefficient of the quotient of two functions?	K1	CO1
b)	Write the Leibnitz formula for the nth derivative of a product.	K1	CO1
c)	What is the first order partial differential coefficients of $u = \log \frac{x^2+y^2}{xy}$	K1	CO1
d)	Mention any two types of integration.	K1	CO1
e)	If u and v are functions of x, then define the integration of uv with respect to x.	K1	CO1

2.	**Choose the correct answer**	**(5 x 1 = 5)**	
a)	The differential coefficient of $\cos(ax + b)$ is _____ (a) $a\sin(ax + b)$ (c) $-a\sin(ax + b)$ (b) $a\cos(ax + b)$ (d) $-a\cos(ax + b)$	K1	CO1
b)	How many points of inflexion is possible for the curve $y = 3x^2 - 2x^3$? (a) 3 (b) 1 (c) 2 (d) none	K1	CO1
c)	$u = \frac{xy}{x+y}$ is a homogenous function of x and y of degree _____. (a) 1 (b) 2 (c) 3 (d) 6	K1	CO1
d)	$\int \frac{dx}{a^2-x^2} =$ _____. (a) $\frac{1}{2a}\log\frac{x-a}{x+a}$ (c) $\frac{1}{2a}\log\frac{a-x}{a+x}$ (b) $\frac{1}{2a}\log\frac{x+a}{x-a}$ (d) $\frac{1}{2a}\log\frac{a+x}{a-x}$	K1	CO1
e)	Find $\int_1^2 x^2 dx$ is _____. (a) $\frac{7}{3}$ (b) $\frac{8}{3}$ (c) $\frac{5}{3}$ (d) $\frac{11}{3}$	K1	CO1

3.	**Fill in the blanks**	**(5 x 1 = 5)**	
a)	The differential coefficient of $\sec x$ is _____.	K2	CO1
b)	The range of x for the function $2x^3 - 15x^2 - 84x + 7$ to be a decreasing function is _____.	K2	CO1

c)	If $u = x \sin y$ then $\frac{\partial^2 u}{\partial x^2}$ is _____.	K2	CO1
d)	Integrating $\frac{dx}{4-9x^2}$ with respect to x is _____.	K2	CO1
e)	The reduction formula for the function $x^n \cos ax$ is _____.	K2	CO1
4.	**True or False** **(5 x 1 = 5)**		
a)	Differentiation of e^t with respect to $\sqrt{t}$ is $2\sqrt{t} \cdot e^t$.	K2	CO1
b)	The function $y = 3x^2 - 2x^3$ is concave upwards for the points $x > \frac{1}{2}$.	K2	CO1
c)	A function $f(x, y)$ attains a minimum value if $rt - s^2 > 0$ and $r > 0$.	K2	CO1
d)	The integration of the function $x^2 \cos(x^3)$ with respect to x is $\sin(x^3)$.	K2	CO1
e)	$\int_0^{\pi/2} \cos^8 x \, dx = \frac{35\pi}{256}$.	K2	CO1

SECTION B

Answer any TWO of the following		**(2 x 10 = 20)**	
5.	(a) Discover the differential coefficient of $\log(\tan e^x)$ with respect to x. (b) Solve $\frac{d}{dx}\left(\frac{x^3}{3x-2}\right)$.	K3	CO2
6.	If $x(1+y)^{1/2} + y(1+x)^{1/2} = 0$, prove that $\frac{dy}{dx} = -\frac{1}{(1+x^2)}$.	K3	CO2
7.	Determine y_n, when $y = \frac{x^2}{(x+1)^2(x+2)}$.	K3	CO2
8.	Apply the appropriate property of definite integral and calculate $\int_0^{\pi/2} \log \sin x \, dx$.	K3	CO2

SECTION C

Answer any TWO of the following		**(2 x 10 = 20)**	
9.	By applying logarithmic differentiation, differentiate $x^4 \frac{(x^2+4)^{1/3}}{(4x^2-7)^{1/2}}$ with respect to x.	K4	CO3
10.	Estimate the maxima and minima points of the function $x^5 - 5x^4 + 5x^3 + 10$.	K4	CO3
11.	State Euler's theorem and analyse whether it is true for the function $u = x^3 - 3x^2y + 3xy^2 + y^3$.	K4	CO3
12.	Integrate $\frac{3x-2}{\sqrt{4x^2-4x-5}}$ with respect to x.	K4	CO3

SECTION D

Answer any ONE of the following		**(1 x 20 = 20)**	
13.	Justify the equations $(1 - x^2)y_2 - xy_1 + m^2 y = 0$ and $(1 - x^2)y_{n+2} - (2n + 1)xy_{n+1} + (m^2 - n^2)y = 0$, if $y = \sin(m \sin^{-1} x)$.	K5	CO4
14.	Evaluate $\int \frac{3x+1}{(x-1)^2(x+3)} dx$, using partial fractions.	K5	CO4

SECTION E

Answer any ONE of the following (1 x 20 = 20)

15.	Derive the maximum and minimum values of the curve $u = x^3 + 3xy^2 - 15x^2 - 15y^2 + 72x$.	K6	CO5
16.	Explain the five properties of definite integration and hence show that $\int_0^\pi \theta \sin^3 \theta = \frac{2\pi}{3}$.	K6	CO5

$$$$$$$